

Population Analysis of (*R*)-Thiazolidine-4-carboxylic Acid

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The structure of (*R*)-thiazolidine-4-carboxylic acid was refined at 111 and 293 K, respectively. Population analysis, which refines the population of the electrons in the molecule, was carried out, the result being compared with that of theoretical calculation.

In studies on structure by means of X-ray diffraction, the recent development of the diffractometer has afforded intensity data accurate enough for discussion of the distribution of the electron cloud. The population analysis¹⁾ deals with the observed electron density distribution quantitatively. The number of electrons in each non-bonded and bonded orbital is derived from the refinement, from which bond orders and charge distribution of the molecule can be discussed.

(*R*)-Thiazolidine-4-carboxylic acid is an analogue of the proline and cysteine, and the structure was determined by Chacko.²⁾ In this paper, the structure of this molecule at both 293 and 111 K was refined by the population analysis method, the electronic structure being discussed in comparison with the theoretical one.

Experimental

Two sets of intensity data were measured at 293 and 111 K with a four circle diffractometer. Each set has 4 octants of the reciprocal space measured in 2θ - θ mode. The crystal data and experimental conditions are given in Tables 1 and 2, respectively. The crystal was treated with liquid nitrogen before the data collection in order to prevent from suffering of the extinction effect. The average structure amplitudes among the equivalent reflections in each set was used for further calculations. The estimated standard deviations, $\sigma(|F_o|)$ of the amplitudes are given by

$$\sigma(|F_o|) = \sqrt{\sum_i \sigma_i(|F_i|)^2 / N},$$

where σ_i denotes the e.s.d. of i -th equivalent reflection.

Structure

The structure was determined independently with the program of automatic structure determination.³⁾ The conventional refinement of the structure was carried out by the full matrix least squares method, applying the following weighting scheme.

$$\text{wt} = c \text{ when } |F_o| = 0$$

$$\text{wt} = 1.0 / (\sigma(|F_o|)^2 + a|F_o| + b|F_o|^2) \text{ when } |F_o| > 0$$

a , b , and c were adjusted in such a way that the differences between the calculated and observed structure amplitudes are uniform in any region of the reciprocal

TABLE 1. CRYSTAL DATA

Space group	P2 ₁ 2 ₁ 2 ₁	
Z	4	
Lattice constant	293 K	111 K
<i>a</i>	9.929±0.002 Å	9.813±0.002 Å
<i>b</i>	9.936±0.002 Å	9.885±0.002 Å
<i>c</i>	5.664±0.001 Å	5.663±0.001 Å
<i>d</i> _{calcd} (g/cm ³)	1.58	1.61
<i>d</i> _{obsd} (g/cm ³)	1.57	
Linear absorption coefficient μ (cm ⁻¹)	4.947	4.855

TABLE 2. EXPERIMENTAL CONDITIONS

Crystal size	0.1×0.1×0.2 mm ³	0.2×0.2×0.4 mm ³
Source	Mo K α	Mo K α
Scan speed (2 θ deg./min)	2	4
Background (s)	10	10
2 θ _{max} (deg.)	70	70
Monochromator	Graphite (002) plane	
Temperature	111±3 K	293±3 K

TABLE 3. DETERMINATION OF THE ABSOLUTE CONFIGURATION (293 K)

	<i>R</i> config.	<i>S</i> config.
f''	-0.16	+0.16
<i>R</i> (%)	3.95	4.06
<i>R</i> _w (%)	2.27	2.35

space. The absolute configuration of the molecule was also determined to be the *R*-configuration from the anomalous scattering of the sulfur atom (Table 3).

After the refinement with all reflections, the structure was refined using the higher order reflections only at 111 K ($\sin \theta/\lambda \geq 0.65$, hereafter HOR) to reduce the effect of the bonding electrons. In comparison with the case of the low-temperature refinement based on all the reflections, all the positional parameters showed no significant shifts, the values of the temperature factors becoming smaller than those of all the reflections as expected (Table 5).

The bond lengths and angles, shown in Fig. 1, are essentially the same as those reported by Chacko.

Difference Fourier Synthesis

The difference Fourier synthesis was carried out with the low and high temperature data phased by the

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final parameters of HOR (Figs. 2 and 3). Some density humps probably due to the bonding electrons are found on the C_3 -S- C_4 plane. Peaks A and B are situated off the bonds S- C_3 and S- C_4 , respectively, in particular deviating from the S- C_4 bond direction. This was found in both the high and low temperature maps, the angles of A-S-B being 105 and 97°, respectively. This indicates that the 3p atomic orbital of the sulfur atom is hybridized with the 3s orbital, the humps due to the bonding electrons deviating from the interatomic vectors as seen in cyclopropane.⁵⁾ The

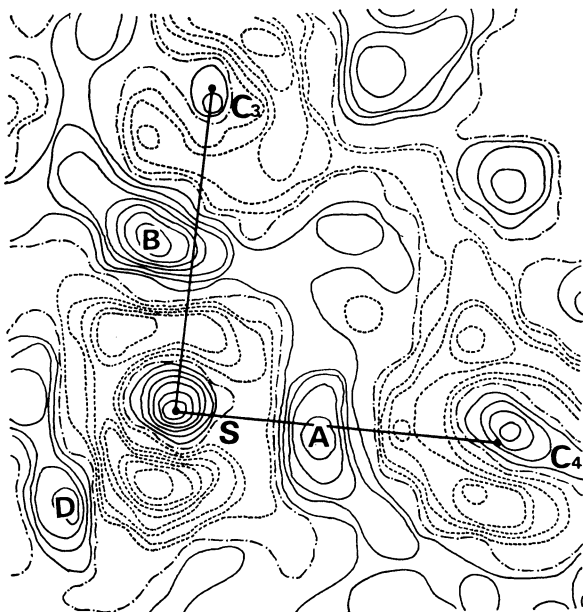


Fig. 2. Difference electron density map in the C₃-S-C₄ plane (111 K). Contours are drawn at the interval of 0.05 e/Å³. The solid lines are above 0 e/Å³ and the broken lines are below 0 e/Å³.

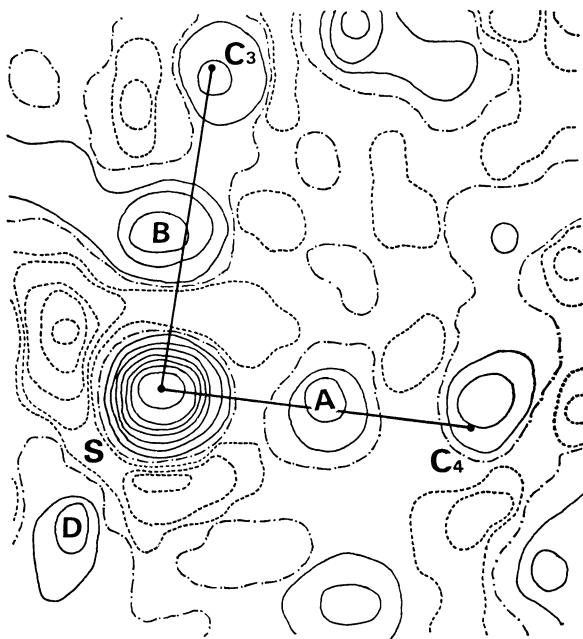


Fig. 3. Difference electron density map in the C₃-S-C₄ plane (293 K).

$$\rho = \sum_i^{\text{occupied}} \Psi_i^* \Psi_i \quad (1)$$

The molecular orbital is represented as a linear combination of the atomic basis function ϕ_μ

$$\Psi_i = \sum_\mu C_{\mu i} \phi_\mu \quad (2)$$

Substituting Eq. 2 into Eq. 1, we have

$$\begin{aligned} \rho &= \sum_i^{\text{occupied}} \sum_{\mu, \nu} C_{\mu i}^* C_{\nu i} \phi_\mu^* \phi_\nu \\ &= \sum_{\mu, \nu} P_{\mu \nu} \phi_\mu^* \phi_\nu \end{aligned} \quad (3)$$

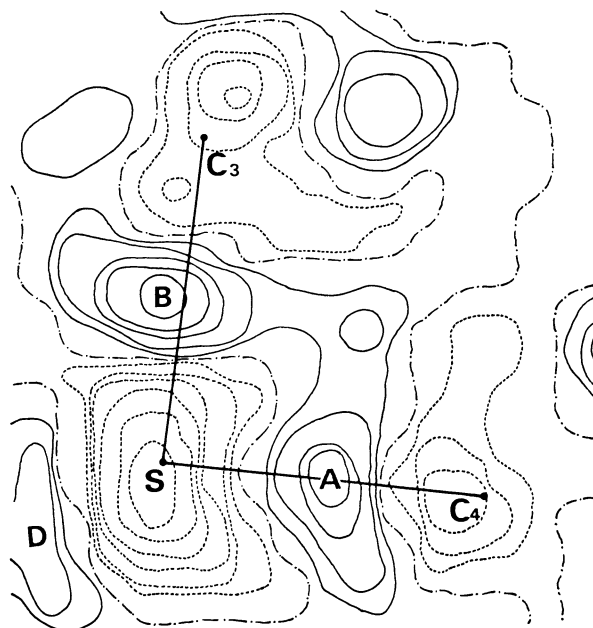


Fig. 4. Difference electron density map in the C₃-S-C₄ plane using only low order reflections ($\sin\theta/\lambda < 0.65$) at 111 K.

$$P_{\mu \nu} = \sum_i^{\text{occupied}} C_{\mu i}^* C_{\nu i} \quad (4)$$

where $P_{\mu \nu}$ is a population parameter. The structure factor of the crystal is represented by

$$F_h = \sum_{j, k} F[\rho_k] \cdot T_{kj} \cdot \exp(i\mathbf{s} \cdot \mathbf{r}_{kj}) \quad (5)$$

j : atom

k : symmetry operation

$F[\rho_k]$: molecular transform

T_{kj} : temperature factor

where

$$\begin{aligned} F[\rho] &= \sum_{\mu, \nu} P_{\mu \nu} F[\phi_\mu^* \phi_\nu] \\ &= \sum_{\mu, \nu} P_{\mu \nu} \int \phi_\mu^* \phi_\nu \exp(i\mathbf{s} \cdot \mathbf{r}) d\mathbf{r} \\ &= \sum_{\mu, \nu} P_{\mu \nu} X_{\mu \nu} \end{aligned} \quad (6)$$

$X_{\mu \nu}$ is called the generalized structure factor. In the population analysis developed by Stewart,¹⁾ the structure factor, F_h , is separated into two parts, $F_h(\text{atom})$ and $F_h(\text{bond})$, the former consisting of the atomic orbitals from one atom only, and the latter showing the effect of the overlap between orbitals of the bonded atoms. As a whole, considering the anomalous effects, F_h is written as follows.

$$F_h = F_h(\text{atom}) + F_h(\text{bond}) = A + iB, \quad (7)$$

$$\begin{aligned} A &= \sum_{j, k} [(f_{\text{core}, j} + f'_j + P_{vj} f_{vj} - 3f_{\text{np}, \text{np}, j} Q_{kj}) \cdot \\ &\quad \cos(2\pi h r_{kj}) \cdot T_{kj} - (f''_j + \sqrt{3} f_{\text{ns}, \text{np}, j} D_{kj}) \cdot \\ &\quad \sin(2\pi h r_{kj}) \cdot T_{kj}] + \sum_m b_m f_{\text{bond}, m} \sum_k \cos(2\pi h r_{km}) \cdot T_{km}, \\ B &= \sum_{j, k} [(f_{\text{core}, j} + f'_j + P_{vj} f_{vj} - 3f_{\text{np}, \text{np}, j} Q_{kj}) \cdot \\ &\quad \sin(2\pi h r_{kj}) \cdot T_{kj} - (f''_j + \sqrt{3} f_{\text{ns}, \text{np}, j} D_{kj}) \cdot \\ &\quad \cos(2\pi h r_{kj}) \cdot T_{kj}] + \sum_m b_m f_{\text{bond}, m} \sum_k \sin(2\pi h r_{km}) \cdot T_{km}, \end{aligned}$$

$$f_{\text{core}} = 2X_{1s, 1s} \text{ or } 2X_{1s, 1s} + 2X_{2s, 2s} + 6X_{2p, 2p},$$

$$f_v = aX_{ns,ns} + bX_{np_x,np_x} + cX_{np_y,np_y} + dX_{np_z,np_z}$$

- a, b, c, d : the scattering of electrons in each orbital
 $f_{ns,np}$: the scattering effect due to the overlap between ϕ_{ns} and ϕ_{np}
 $f_{np,np}$: the scattering effect due to the overlap between ϕ_{np_x} and ϕ_{np_y} , or ϕ_{np_y} and ϕ_{np_z} , or ϕ_{np_z} and ϕ_{np_x}
 f_{bond} : the scattering factor of the postulated orbital at the center of the bond (r_m)

TABLE 6. POPULATION ANALYSIS REFINEMENT (111K)

	R_w^a	Scale of $ F_o $
ELS	2.28	0.839
OCR	2.07	0.841
TCR	2.06	0.841
b)	2.76	0.819
HOR	5.26	0.895

a) $R_w = (\sum w||F_o| - |F_c|| / \sum |F_o|) \times 100.0$. b) Value obtained by the usual refinement.

TABLE 7. EXPERIMENTAL AND THEORETICAL POPULATION PARAMETERS

	111 K			293 K				111 K			293 K		
	ELS	OCR	TCR	ELS	OCR	CNDO		ELS	OCR	TCR	ELS	OCR	CNDO
Sp _v	6.908	6.990	7.174	6.233	6.202	6.011	C ₂ p _v	3.794	3.783	3.792	3.872	3.839	4.008
d _x		-0.817	-0.607		1.006	-0.211	d _x		-0.063	0.094		0.311	0.022
d _y		-0.042	0.159		0.486	-0.269	d _y		0.556	0.709		0.375	0.073
d _z		-0.444	-0.842		-0.703	-0.232	d _z		-0.362	-0.573		-0.207	-0.196
q ₁		-0.112	-0.095		0.015		q ₁		-0.015	-0.046		0.052	
q ₂		-0.287	-0.210		0.015		q ₃		-0.199	-0.170		0.143	
q ₃		0.008	-0.069		-0.014	-0.159	q ₃		-0.109	-0.143		0.345	-0.013
q ₄		0.049	0.016		0.143	0.172	q ₄		-0.114	-0.113		-0.206	0.227
q ₅		0.232	0.194		0.122	-0.424	q ₅		-0.304	-0.289		0.032	-0.116
O ₁ p _v	6.158	6.139	5.935	6.247	6.245	6.492	C ₃ p _v	3.818	3.744	3.624	3.806	3.775	3.956
d _x		-0.023	-0.049		0.159	0.231	d _x		-0.121	-0.050		0.239	0.053
d _y		0.114	0.171		0.244	-0.086	d _y		0.143	0.192		0.377	-0.018
d _z		-0.322	-0.415		-0.137	-0.361	d _z		-0.245	-0.436		-0.183	-0.024
q ₁		0.083	0.158		0.007		q ₁		-0.109	-0.052		-0.002	
q ₂		0.061	0.084		0.021		q ₂		-0.047	-0.024		0.084	
q ₃		-0.176	-0.035		-0.029	-0.085	q ₃		-0.996	-0.006		0.183	-0.051
q ₄		-0.283	-0.428		0.030	0.249	q ₄		-0.295	0.145		-0.019	-0.004
q ₅		-0.181	-0.215		0.078	-0.162	q ₅		0.219	0.093		0.060	0.012
O ₂ p _v	6.121	6.066	5.967	6.258	6.240	6.519	C ₄ p _v	3.501	3.458	3.558	3.596	3.635	3.927
d _x		0.020	0.070		0.107	-0.324	d _x		-0.093	0.005		0.128	-0.056
d _y		0.115	0.136		0.278	0.287	d _y		0.034	0.124		0.156	-0.115
d _z		-0.369	-0.536		-0.154	0.039	d _z		-0.170	-0.350		0.033	-0.005
q ₁		0.086	0.082		0.083		q ₁		-0.002	-0.038		0.067	
q ₂		0.012	0.105		0.103		q ₂		0.209	0.131		0.194	
q ₃		-0.043	-0.033		0.120	0.193	q ₃		-0.141	-0.057		0.209	-0.094
q ₄		-0.207	-0.258		-0.143	-0.005	q ₄		-0.161	-0.034		-0.075	0.007
q ₅		-0.108	-0.084		0.071	0.083	q ₅		0.128	0.092		-0.078	0.004
Np _v	5.077	5.009	5.092	5.079	5.040	4.978	H ₁	1.039	1.046	1.044	1.083	1.105	0.963
d _x		-0.274	-0.235		0.047	0.039	H ₂	1.038	1.078	1.048	1.047	1.076	0.941
d _y		-0.077	-0.043		0.223	-0.019	H ₃	1.010	0.999	1.006	1.074	1.067	0.990
d _z		-0.241	-0.362		-0.265	-0.055	H ₄	1.118	1.152	1.158	1.119	1.133	0.962
q ₁		-0.010	0.029		0.074		H ₅	0.959	0.957	0.975	1.054	1.016	0.973
q ₂		0.054	0.124		0.066		H ₆	0.967	0.990	1.026	0.940	0.992	0.805
q ₃		-0.146	-0.310		0.010	0.002	H ₇	0.862	0.962	0.903	0.972	0.998	0.860
q ₄		-0.084	-0.175		-0.136	-0.088	S-C ₃			-0.664			0.559
q ₅		0.171	0.132		0.104	0.004	S-C ₄			-0.279			0.588
C ₁ p _v	3.626	3.627	3.343	3.622	3.637	3.615	C ₁ -O ₁			1.131			0.841
d _x		0.155	0.220		0.269	-0.026	C ₁ -O ₂			0.768			0.771
d _y		-0.236	-0.243		0.377	-0.051	C ₁ -C ₂			0.781			0.563
d _z		-0.318	-0.570		-0.479	-0.094	C ₂ -N			-1.057			0.547
q ₁		-0.174	-0.063		-0.060		C ₄ -N			1.320			0.562
q ₂		0.328	0.358		0.174		C ₂ -C ₃			0.615			0.726
q ₃		-0.827	-0.671		0.343	-0.060							
q ₄		0.056	-0.095		0.107	-0.066							
q ₅		0.580	0.514		-0.219	0.041							

D_{kj} and Q_{kj} in Eqs. are expressed in terms of parameters d and q in this table.

$$D_{kj} = d_{xj}S_{xsf} + d_{yj}S_{ysf} + d_{zj}S_{zsf}$$

$$Q_{kj} = q_{1j}(S_{xsf}^2 - S_{ysf}^2) + q_{2j}(S_{zsf}^2 - 1/3) + q_{3j}S_{xsf}S_{ysf} + q_{4j}S_{xsf}S_{zsf} + q_{5j}S_{ysf}S_{zsf}$$

S_{xsf} , S_{ysf} , and S_{zsf} are direction cosines of the scattering vector to the principal axes of the p_x , p_y , and p_z orbitals of the j -th atom, respectively.

TABLE 8. EXPERIMENTAL AND THEORETICAL CHARGES

Atom	111 K	293 K	CNDO	EN ^{a)}
S	-0.990	-0.202	-0.011	0.00
O ₁	-0.139	-0.245	-0.492	—
O ₂	-0.066	-0.240	-0.519	—
N	-0.009	-0.040	0.022	0.48
C ₁	0.373	0.363	0.385	0.40
C ₂	0.217	0.161	-0.008	0.02
C ₃	0.256	0.225	0.044	-0.14
C ₄	0.542	0.365	0.073	-0.03
H ₁	-0.046	-0.105	0.037	0.07
H ₂	-0.078	-0.076	0.059	0.07
H ₃	0.001	-0.067	0.010	0.07
H ₄	-0.152	-0.133	0.038	0.07
H ₅	0.043	-0.016	0.027	0.07
H ₆	0.010	0.008	0.195	0.17
H ₇	0.038	0.002	0.141	0.17

a) Calculated from the differences in electronegativity of the neighboring atoms with Mulliken's equation.

The parameter P_v is the number of valence electrons associated with the j -th atom. A dipole distortion of these spherical valence electrons is described by D and a quadrupole distortion by Q . The extended L-shell method(ELS) refines P_v after the usual refinement of the positional and thermal parameters. P , Q , and D are treated as adjustable parameters in the one-center refinement(OCR). The two-center refinement (TCR) includes the b_{mk} term as variables in addition to those of OCR.

In the present calculation, the $f_{core,j}$'s of the carbon and nitrogen atoms are taken from those calculated by use of Clementi's analytical SCF atomic orbitals.⁶⁾ f_{vj} 's of these atoms being taken from these calculated by use of the Slater-type atomic orbitals.⁷⁾ f_v of the hydrogen atom is taken from Stewart-Davidson-Simpson's value,⁸⁾ f_{core} and f_v of the sulfur atom are from the Roothaan-Hartree-Fock wave function by Fukamachi⁹⁾ and $f_{ns,np}$ and $f_{np,np}$ are those of Stewart.¹⁰⁾ In the case of TCR, f_{bond} is taken as $\overline{X_{ns,ns}(s \cdot r)}$ at the center of each bond where $\overline{X_{ns,ns}(s \cdot r)}$ is a spherically averaged value of the overlapping function between the ns-orbital. The population analysis of the present molecule at 111 and 293 K was carried out after the usual refinement, using ELS, OCR, and TCR, successively. The computer program was written by one of the authors (J.K.). The positional and thermal parameters of OCR and TCR are fixed to the values of ELS. The R factors and scale factors after the refinement are given in Table 6. The R factor decreased from that of the usual refinement, the scale factor of $|F_o|$ becoming close to that of HOR.

The population parameters, thus refined, are compared with the theoretical values calculated by the CNDO method (Table 7). Experimental and theoretical charges are given in Table 8. The experimental values show as a whole the same distribution as the theoretical values except for the hydrogen atoms. However, the charge of the sulfur atom in the low-temperature data deviates considerably from that of CNDO and EN, because of errors in $|F_o|$'s and in-

accuracy in the scattering factor of the sulfur atom.

The dipole moment of the molecule can be calculated from the charge distribution in the density map. The absolute values of the dipole moment of the molecule, 11.4 Debye at 111 K and 9.3 Debye at 293 K are roughly in line with the theoretical value (13.2 Debye). However, their directions do not agree with each other.

Discussion

In the present work, we used the higher order reflections refinement to reduce the effect of the spread of the electron cloud. The effect, however, still remains in the parameters, in spite of the considerable depression of the temperature factor (Table 5). The population analysis seems to refine the electronic structure of the molecule; the overall scale factor is improved so as to become close to the value of the HOR, the R factor decreasing significantly from the usual refinement. Some discrepancy in distribution is found between the X-ray and theoretical results. In particular, the sulfur atom at 111 K has a more negative charge than that at 293 K and CNDO. This may be explained as follows:

1) The population analysis treats the small differences between the conventionally calculated structure factors and the observed. The data at 111 K have four times weaker intensities than those at 293 K, depending on the size of the crystal used. Though the smaller crystal gives less systematic errors due to absorption and extinction in the data, the experimental errors in the weaker intensities at 111 K may give rise to some discrepancies.

2) The discrepancy may be due to the neglect of the d-orbital of the sulfur atom and the different character between the L- and M-shell orbitals which are not distinguished in the present analysis.

In spite of these discrepancies we can say that the population analysis throws light on the character of the molecule.

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